

An Investigation of Equivalence between the Bulk-based and the Brane-based Approaches for Anisotropic Models

Gülçin Uluyazı

Physics Department, İstanbul University, Vezneciler, Turkey.

(Dated: 17.12.2011)

We investigate the relation between the brane-based and the bulk-based approaches for anisotropic case in brane-world models. In the brane-based approach, the brane is chosen to be fixed on a coordinate system, whereas in the bulk-based approach it is no longer static as it moves along the extra dimension. It was shown that these two approaches are basically equal for specific models [1], [2]. In this paper, it is aimed to get general formalism of the equivalence obtained in Mukohyama et al. [1]. We found that calculations driven by a general anisotropic bulk-based metric yield a brane-based metric in Gaussian Normal Coordinates by conserving spatial anisotropy. We also derive solutions for an anisotropic bulk-based model and apply to get corresponding brane-based metric of the model.

PACS numbers:

I. INTRODUCTION

Although extra dimensions have been widely involved in the String Theory for several decades, their introduction into cosmology has been recently offered to give provide a new perception of the Universe and its evolution. It was proposed [3] that our Universe may be a 3- dimensional surface (called domain wall or brane) embedded in a higher dimensional space (called bulk). Following the novel approach to hierarchy problem proposed by Arkani-Hamed, Dimopoulos and Dvali [4]-[6], Randall and Sundrum [7],[8] suggested that the brane representing our universe lies in a 5-dimensional anti-de-Sitter (AdS) bulk, which is strongly curved in order to ensure that gravitation has effectively 4- dimensional character on the brane, even if the extra dimension is finite.

The key feature of the Randall Sundrum (RS) models is that the induced metric on the brane is flat Minkowski spacetime. The brane spacetime is generally static owing to the effect of brane energy and the tension is neutralized in the presence of bulk cosmological constant. However our universe is not Minkowskian and has an expansion. To allow time dependent expansion along spatial directions in a brane-world model, some authors [9] - [20] consider the "brane-based approach" in which the brane is fixed at a point along the extra dimension. Although the cosmology of the brane is more apparent in this approach, bulk spacetime structure remains less transparent.

The brane based approach is usually handled in Gaussian Normal Coordinates, in which the metric component of extra dimension is normal to the brane. Writing 5-dimensional Einstein Field Equations

$$R_{AB}^5 - \frac{1}{2}g_{AB}^5 = -\Lambda_5 g_{AB}^5 + \kappa_5^2 T_{AB}^5 \quad (1)$$

together with Gauss-Codacci equations

$$R_{ABCD}^4 = h_A^E h_B^F h_H^G h_D^E R_{EFGH}^5 + K_{AC} K_{BD} - K_{AD} K_{BC} \quad (2)$$

$$\nabla_B^4 K_A^B - \nabla_A^4 K = h_A^B R_{BC}^5 n^C \quad (3)$$

and Israel Junction Conditions,

$$[h_{AB}] = 0 \quad (4)$$

$$[K_{AB}] = \kappa_5^2 \left(S_{AB} - \frac{1}{3} h_{AB} S \right) \quad (5)$$

cosmological properties of brane can be found. Here T_{AB}^5 represents any 5-dimensional energy-momentum of the gravitational sector and it provides a conservation equation $\nabla_A T^{AB} = 0$. S_{AB} and K_{AB} represent energy momentum tensor of brane and extrinsic curvature tensor respectively, so that S and K are their traces. n^A being the unit vector normal to the brane, $h_{AB} = g_{AB} - n_A n_B$ is the induced metric on the brane.

On the other hand, using induced field equations is a more elegant way for the derivation of the brane world cosmological equations. Shiromizu-Maeda-Sasaki [21] obtained 4-dimensional induced field equations by projecting 5-dimensional equations onto the brane.

$$G_{AB}^4 = -\Lambda_4 h_{AB} + 8\pi G_5 \tau_{AB} + \kappa_5^4 \pi_{AB}^5 - E_{AB}^5 \quad (6)$$

$$\Lambda_4 = \frac{1}{2} \kappa_5^2 [\Lambda_5 + \frac{1}{2} \kappa_5^2 \lambda^2] \quad (7)$$

$$G_5 = \frac{\kappa_5^4 \lambda}{48\pi} \quad (8)$$

$$\pi_{AB}^5 = -\frac{1}{4} \tau_{AC} \tau_B^C + \frac{1}{12} \tau \tau_{AB} + \frac{1}{8} h_{AB} \tau_{CD} \tau^{CD} - \frac{1}{24} h_{AB} \tau^2 \quad (9)$$

These equations are also important to include bulk effects in the 4-dimensional spacetime.

Apart from the brane-based approach, the bulk-based approach accepts a moving brane (or a domain wall) following some timelike trajectories in a static bulk spacetime. It was first obtained by Ida [22] in which the most general static AdS solution

$$ds^2 = h(r) dt^2 - h^{-1}(r) dr^2 - r^2 \left[\frac{d\chi^2}{1 - \kappa\chi^2} + \chi^2 d\Omega_{II}^2 \right] \quad (10)$$

where $h(r) = \kappa - \frac{\mu}{r^2} + k^2 r^2$.

The bulk-based approach is quite general and does not depend on Z_2 symmetry around the brane. The cosmological solution of the brane will be described by its movement in the bulk. Some other papers using this approach are given in [23]-[27].

It was found by Mukohyama et. al. [1] that these two approaches are equivalent. They have constructed a coordinate transformation relating Ida's solution [22] to Binetruy et. al. [9]. In contrast with [1], an attempt by Bowcock et. al. [2] which starts from brane-based approach has also given the same result.

Our aim in this paper is, to construct a general formalism for coordinate transformation between the bulk-based and the brane-based approaches and then investigating their equivalence in the case of anisotropic models. Observations at angular distribution of extra-galactic radio sources, spatial distribution of the redshifts of extra-galactic objects and temperature distribution of cosmic microwave radiation confirm that our universe has anisotropy, so it is worth to investigate to anisotropic models to get a more realistic description of the universe. In Sec.II, applying transformation method in [1], we obtain the transformed brane-based metric from the most general bulk-based anisotropic metric ansatz. After obtaining the solution of bulk field equations, we apply the method for a particular anisotropic model in Sec.III and finally Sec.IV is devoted to the summary of the paper and discussions.

II. TRANSFORMATION FROM THE BULK-BASED APPROACH TO BRANE-BASED ONE

The most general 5-dimensional anisotropic bulk-based metric¹ admitting non-zero Killing vector space, which preserves symmetries along congruence lines is

$$ds^2 = -A_0(\hat{r}) d\hat{t}^2 + A_{ij}(\hat{r}) d\hat{x}^i d\hat{x}^j + A_4(\hat{r}) d\hat{r}^2 \quad (11)$$

where \hat{r} denotes the extra spatial coordinate and $i, j = 1, 2, 3$ are indices of 3-dimensional spacetime. The 4-dimensional brane, corresponding to our universe moves along the extra dimension and is described by (τ, x^i) coordinates. Base vectors and one forms, in the 5-dimensional and 4-dimensional spacetime, respectively are given in below.

$$e_A \equiv \partial_A = \frac{\partial}{\partial \hat{x}^A} = (\partial_{\hat{t}}, \partial_{\hat{x}^i}, \partial_{\hat{r}}) \omega^A \equiv d\hat{x}^A = (d\hat{t}, d\hat{x}^i, d\hat{r}) \quad (12)$$

¹ Although the most general anisotropic metric has non-diagonal form for every components, it becomes (11) in the case of providing non-zero Killing field requirement.

$$e_\mu \equiv \partial_\mu = \frac{\partial}{\partial \hat{x}_\mu} = (\partial_\tau, \partial_{x^i}) \omega^\mu \equiv dx^\mu = (d\tau, dx^i) \quad (13)$$

Here we define $A = 0.4$, $\mu = 0.3$. The brane represented by $\hat{r} = R(\hat{t})$ hypersurfaces can be induced on 4-dimensional spacetime via the following transformations.

$$\begin{aligned} \hat{t} &= T(\tau) \rightarrow d\hat{t} = \dot{T} d\tau \\ \hat{x}^i &= x^i \rightarrow d\hat{x}^i = dx^i \\ \hat{r} &= R(\tau) \rightarrow d\hat{r} = \dot{R} d\tau \end{aligned} \quad (14)$$

The induced metric on brane is then

$$ds_{brane}^2 = -(A_0 \dot{T}^2 - A_4 \dot{R}^2) d\tau^2 + A_{ij}(R(\tau)) dx^i dx^j \quad (15)$$

where we introduced cosmological time τ and cosmological scale factor $R(\tau)$. The dot denotes the derivative respect to τ . Now we can construct a vector space generated by tangent vectors of geodesics intersecting with hypersurface $\hat{r} = R(\hat{t})$ perpendicularly

$$u^A = e_\tau^A \partial_A = \dot{T} \partial_{\hat{t}} + \dot{R} \partial_{\hat{r}} \quad (16)$$

We choose geodesics as spacelike and having zero \hat{x}^i -components to provide a timelike hypersurface. The Killing field of bulk spacetime helps us to find constants of motion along geodesics:

$$g_{AB} u^A \xi^B = -E \quad (17)$$

$$g_{AB} u^A u^B = 1 \quad (18)$$

where E is an constant of integration. Using tangent vector's components in (16), we obtain

$$u^A = \left(\frac{E}{A_0}, 0, 0, 0, \mp \frac{A_0 + E^2}{A_0 A_4} \right) \quad (19)$$

The trajectory of the geodesic is given by

$$\frac{dx^A}{dw} = u^A \quad (20)$$

where w is the affine parameter. All points (P) on the hypersurface described by (τ, x^i) coordinates intersecting perpendicularly with an affinely parameterized geodesic. Hence we can describe the point P , with a new coordinate set (τ, x^i, w) , where the new coordinate w is now an extra spatial coordinate of P and this system is called brane-based coordinates. One can easily construct the brane-based metric, from the bulk-based one by applying transformations: $\hat{r} = \hat{r}(\tau, w)$, $\hat{t} = \hat{t}(\tau, w)$

$$d\hat{t} = \left(\frac{\partial \hat{t}}{\partial \tau} \right) d\tau + \left(\frac{\partial \hat{t}}{\partial w} \right) dw = e_{\hat{\tau}}^{\hat{t}} d\tau + e_w^{\hat{t}} dw \quad (21)$$

$$d\hat{r} = \left(\frac{\partial \hat{r}}{\partial \tau} \right) d\tau + \left(\frac{\partial \hat{r}}{\partial w} \right) dw = e_{\hat{\tau}}^{\hat{r}} d\tau + e_w^{\hat{r}} dw \quad (22)$$

Substituting them in (11) gives

$$\begin{aligned} ds^2 &= - \left[A_0 \left(\frac{\partial \hat{t}}{\partial \tau} \right)^2 - A_4 \left(\frac{\partial \hat{r}}{\partial \tau} \right)^2 \right] d\tau^2 + A_{ij} dx^i dx^j \\ &\quad + \left[-A_0 \left(\frac{\partial \hat{t}}{\partial w} \right)^2 + A_4 \left(\frac{\partial \hat{r}}{\partial w} \right)^2 \right] dw^2 \\ &\quad + 2 \left[-A_0 \left(\frac{\partial \hat{t}}{\partial \tau} \right) \left(\frac{\partial \hat{t}}{\partial w} \right) + A_4 \left(\frac{\partial \hat{r}}{\partial \tau} \right) \left(\frac{\partial \hat{r}}{\partial w} \right) \right] d\tau dw \end{aligned} \quad (23)$$

The final metric in (23) is a general form of brane-based metric which is transformed from the bulk-based metric. We need to find the exact forms of transformation coefficients denoted by partial derivatives in (23). Replacing (19) into (20) we get two equations

$$u^{\hat{t}} = e^{\hat{t}}_w = \frac{\partial \hat{t}}{\partial w} = \frac{E}{A_0} \quad (24)$$

$$u^{\hat{r}} = e^{\hat{r}}_w = \frac{\partial \hat{r}}{\partial w} = \mp \sqrt{\frac{A_0 + E^2}{A_0 A_4}} \quad (25)$$

of which the second one gives an integral relation between extra coordinates of the two approaches. In the case the components of the bulk-based metric are known, (25) can be solved exactly.

$$\mp w + w_0(\tau) = \int \frac{d\hat{r}}{\sqrt{\frac{A_0 + E^2}{A_0 A_4}}} \quad (26)$$

On the other hand, we get transverse coefficients in (24) and (25) from $dw/dx^B = g_{AB}u^A$

$$e_{\hat{t}w} = \frac{\partial w}{\partial \hat{t}} = -E \quad (27)$$

$$e_{\hat{x}^i w} = \frac{\partial w}{\partial \hat{x}^i} = 0 \quad (28)$$

$$e_{\hat{r}w} = \frac{\partial w}{\partial \hat{r}} = \pm \sqrt{\frac{A_4(A_0 + E^2)}{A_0}} \quad (29)$$

The integrability condition $ddw = 0$ is equivalent to

$$\left(\frac{\partial w}{\partial \hat{t}}\right) d\hat{t} = -\left(\frac{\partial w}{\partial \hat{r}}\right) d\hat{r} \quad (30)$$

and gives a ratio of coefficients

$$\left(\frac{\partial \hat{t}}{\partial \tau}\right) / \left(\frac{\partial \hat{r}}{\partial \tau}\right) = -\left(\frac{\partial w}{\partial \hat{r}}\right) / \left(\frac{\partial w}{\partial \hat{t}}\right) = \pm \sqrt{\frac{A_4(A_0 + E^2)}{A_0 E^2}} \quad (31)$$

The transverse ratio is then

$$\left(\frac{\partial \tau}{\partial \hat{t}}\right) / \left(\frac{\partial \tau}{\partial \hat{r}}\right) = -\left(\frac{\partial \hat{r}}{\partial w}\right) / \left(\frac{\partial \hat{t}}{\partial w}\right) = \pm \sqrt{\frac{A_0(A_0 + E^2)}{A_4 E^2}} \quad (32)$$

In order to see all transformation coefficients which have been calculated up to now, let us put them into a matrix form as below

$$(\hat{t}, \hat{x}^i, \hat{r}) \rightarrow (\tau, x^i, w) : \begin{pmatrix} e^{\hat{t}}_{\tau} & e^{\hat{t}}_{x^i} & e^{\hat{t}}_w \\ e^{\hat{x}^i}_{\tau} & e^{\hat{x}^i}_{x^i} & e^{\hat{x}^i}_w \\ e^{\hat{r}}_{\tau} & e^{\hat{r}}_{x^i} & e^{\hat{r}}_w \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & \frac{E}{A_0} \\ 0 & 1 & 0 \\ a_{31} & 0 & \pm \sqrt{\frac{A_0 + E^2}{A_4 A_0}} \end{pmatrix} \quad (33)$$

$$(\tau, x^i, w) \rightarrow (\hat{t}, \hat{x}^i, \hat{r}) : \begin{pmatrix} e_{\hat{t}\tau} & e_{\hat{t}x^i} & e_{\hat{t}w} \\ e_{\hat{x}^i\tau} & e_{\hat{x}^i x^i} & e_{\hat{x}^i w} \\ e_{\hat{r}\tau} & e_{\hat{r}x^i} & e_{\hat{r}w} \end{pmatrix} = \begin{pmatrix} a^{11} & 0 & -E \\ 0 & 1 & 0 \\ a^{31} & 0 & \pm \sqrt{\frac{A_4(A_0 + E^2)}{A_0}} \end{pmatrix} \quad (34)$$

Note that, two of the matrix components remain unknown, but they depend on each other according to these equations

$$e_{\hat{\tau}}^{\hat{t}}/e_{\hat{\tau}}^{\hat{r}} = a_{11}/a_{31} = \pm \sqrt{\frac{A_4 (A_0 + E^2)}{A_0 E^2}} \quad (35)$$

$$e_{\hat{t}\tau}/e_{\hat{r}\tau} = a^{11}/a^{31} = \pm \sqrt{\frac{A_0 (A_0 + E^2)}{A_4 E^2}} \quad (36)$$

Substituting partial derivatives from the matrix in (23), we find the final form of the brane-based metric

$$ds^2 = -\frac{A_0(\hat{r}(\tau, w))A_4(\hat{r}(\tau, w))}{E^2} \left(\frac{\partial \hat{r}}{\partial \tau}\right)^2 d\tau^2 + A_{ij}(\hat{r}(\tau, w))dx^i dx^j + dw^2 \quad (37)$$

which is a general form of the brane-based metric in Gaussian Normal Coordinates and it contains spatial anisotropy as expected. If we choose metric components as $A_0 = f(\hat{r})$, $A_4 = 1/f(\hat{r})$ and the 3-dimensional metric as isotropic, Eq. (37) readily yields the result in Ref. [1].

All terms in (34) depend on brane coordinates (τ, w) . The motion constant E of bulk geodesics is not a constant anymore on the brane and turns out to be $E = E(\tau)$. Besides, the term showing partial derivative can be calculated from integral relation (26) after obtaining explicit forms of metric components. For this purpose one must solve vacuum Einstein field equations in the bulk and replace them with the transformed metric. (37) gives the induced metric on brane by setting $w = 0$.

III. APPLICATIONS OF THE METHOD FOR AN ANISOTROPIC BULK-BASED MODEL

To demonstrate an example of the above transformation, we choose the bulk-based metric below

$$ds^2 = -f(\hat{r})d\hat{t}^2 + \hat{r}^2(a(\hat{r})d\hat{x}^2 + b(\hat{r})d\hat{y}^2 + c(\hat{r})d\hat{z}^2) + \frac{1}{f(\hat{r})}d\hat{r}^2 \quad (38)$$

which still conserves spatial anisotropy with three different diagonal terms $a(\hat{r})$, $b(\hat{r})$ and $c(\hat{r})$. Our assumption is diversified from the one in [28] with the factor of dependence of functions' variables. 5-dimensional vacuum Einstein field equations² give two equations for $f(r)$, which is a remarkably important term to find the brane-based metric by transformations given in the previous section.

$$f'' + \frac{f'}{2} \left(\frac{a'}{a} + \frac{b'}{b} + \frac{c'}{c} \right) = \frac{4\Lambda_5}{3} \quad (39)$$

$$\left(\frac{f'}{4} + \frac{f}{\hat{r}} \right) \left(\frac{a'}{a} + \frac{b'}{b} + \frac{c'}{c} \right) + \frac{f}{4} \left(\frac{a' b'}{a b} + \frac{a' c'}{a c} + \frac{b' c'}{b c} \right) + \frac{3f'}{2\hat{r}} + \frac{3f}{\hat{r}^2} = \Lambda_5 \quad (40)$$

Defining p, q, k as constants, we can get an exact solution of $f(r)$ for particular functions

$$a(\hat{r}) = \hat{r}^p, b(\hat{r}) = \hat{r}^q, c(\hat{r}) = \hat{r}^k \quad (41)$$

and equations (39, 40) give rise to

$$f(\hat{r}) = \frac{4\Lambda_5}{3(p+q+k+2)}\hat{r}^2 - C \quad (42)$$

with the condition $p+q+k=6$. Here C is the constant of integration. Then (38) arises

$$ds^2 = -\left(\frac{\Lambda_5}{6}\hat{r}^2 + C \right) d\hat{t}^2 + \hat{r}^2(\hat{r}^p d\hat{x}^2 + \hat{r}^q d\hat{y}^2 + \hat{r}^k d\hat{z}^2) + \left(\frac{\Lambda_5}{6}\hat{r}^2 + C \right)^{-1} d\hat{r}^2 \quad (43)$$

² Field equations are given in the appendix.

as in the form of AdS spacetime with $\Lambda_5 = -\frac{6}{l^2}$. Note that this metric differs from AdS spacetime by its anisotropic spatial section. Substituting $A_0 = f(\hat{r}) = \frac{\Lambda_5}{6}\hat{r}^2 + C$ and $A_4 = 1/f(\hat{r})$ in (26)

$$\mp w + w_0(\tau) = \int \frac{d\hat{r}}{\sqrt{\frac{A_0+E^2}{A_0 A_4}}} = \int \frac{d\hat{r}}{\sqrt{\frac{\Lambda_5}{6}\hat{r}^2 + C + E^2}} \quad (44)$$

we get the relation for two extra coordinates as follows

$$\mp w + w_0(\tau) = \begin{cases} \frac{6}{\Lambda_5} \sinh^{-1} \left(\sqrt{\frac{6}{\Lambda_5}} \frac{\hat{r}}{C+E^2} \right), & C + E^2 > 1 \\ \frac{6}{\Lambda_5} \sin^{-1} \left(\sqrt{\frac{6}{\Lambda_5}} \frac{\hat{r}}{C+E^2} \right), & C + E^2 < 1 \end{cases} \quad (45)$$

Using the fact that geodesics intersect with the hypersurface $r(t_0) = R(t_0(\tau))$ where $w = 0$ at $t = t_0$, we define $w_0(\tau)$ and then get

$$\hat{r}(\tau, w) = \begin{cases} Q_+(\tau) \sinh \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right) + R(\tau) \cosh \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right), & C + E^2 > 1 \\ Q_-(\tau) \sin \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right) + R(\tau) \cos \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right), & C + E^2 < 1 \end{cases} \quad (46)$$

where we represent

$$E = \dot{R}(\tau) \quad (47)$$

$$Q_{\pm}(\tau) = \sqrt{\frac{\Lambda_5}{6} \pm \frac{R(\tau)}{C + \dot{R}^2(\tau)}} [C + \dot{R}^2(\tau)] \quad (48)$$

Calculating partial derivative in (37), we find the corresponding brane-based metric of our anisotropic assumption in (38)

$$ds^2 = -\Phi(\tau, w)d\tau^2 + \hat{r}^2(\tau, w) [\hat{r}^p(\tau, w)dx^2 + \hat{r}^q(\tau, w)dy^2 + \hat{r}^k(\tau, w)dz^2] + dw^2 \quad (49)$$

Here we represent $\Phi(\tau, w)$ as

$$\Phi(\tau, w) = \begin{cases} \frac{1}{HR} \cosh \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right) + \varphi(\tau) \sinh \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right), & C + E^2 > 1 \\ \frac{1}{HR} \cos \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right) + \varphi(\tau) \sin \left(\pm \sqrt{\frac{\Lambda_5}{6}} w \right), & C + E^2 < 1 \end{cases} \quad (50)$$

and

$$\varphi(\tau) = \frac{\frac{4\Lambda_5}{6} (H^2 R^2 + C) (H^2 + \dot{H}) \pm C R^{-1} \pm R (3H^2 + \dot{H})^2}{\sqrt{\frac{\Lambda_5}{6} (C (RH)^{-1} + RH)^2 \pm R (1 + (RH)^{-2})}} \quad (51)$$

H denotes $\dot{R}(\tau)/R$. Here we see that depending on the positive (or negative) values of C , geometry of the Universe will be open (or closed) with hyperbolic (or elliptic) evolution. Metric (49) is completely written in the brane-based coordinates and has the form of Gaussian Normal Coordinates. Additionally it is possible to derive the metric on the brane by setting $w = 0$ as below

$$ds^2 = -\frac{1}{HR} d\tau^2 + R^2(\tau) [R^p(\tau)dx^2 + R^q(\tau)dy^2 + R^k(\tau)dz^2] \quad (52)$$

which is in the form of Kasner-like spacetime.

IV. DISCUSSION

We have studied 5-dimensional brane-world models in the context of two different approaches which are widely used for brane-world cosmologies. One can always build a coordinate transformation linking these approaches under the condition that spacetime symmetries exist, i.e., non-zero Killing vector fields. Although it is possible to constitute a special transformation for each metric respectively, it will certainly be useful to construct a operable transformation procedure. We have first tried to rewrite such a generalized procedure through the formalism presented in [1].

Starting from the most general 5-dimensional bulk-based metric with homogeneous and anisotropic spatial 3-sections (11), we have shown all steps of transformation, keeping metric components nonspecific in order to make any particular metric applicable. One of the most important features of the transformation is the integral equation (26) which relates coordinates of extra dimensions of the two approaches. When metric components are determined from field equations, this equation can be solved and the transformed metric (37) becomes explicit.

It is also interesting that the transformed metric (37) is in the form of Gaussian Normal coordinates. However anisotropy in the spatial sections remains formally unaffected under the transformation and can be obtained exactly from the solution of (26).

In order to employ specific metric components in the transformation, we then derived the solution of a more restricted but anisotropic spacetime (38) allowing analytical calculations of the bulk field equations. Our solution (43) differs from AdS spacetime only with the anisotropic 3-spatial sections, however it is clear that it represents conformally AdS spacetime. Defining $k = \Lambda_5/6$ and setting $C = 1$, \hat{x} , \hat{y} , $\hat{z} = \text{constant}$, 2-dimensional conformal spacetime will be

$$ds^2 = -(k^2 \hat{r}^2 + 1) d\hat{t}^2 + \frac{1}{(k^2 \hat{r}^2 + 1)} d\hat{r}^2 \quad (53)$$

which can also be regarded as a BTZ black hole in the bulk [29].

After performing transformation for the chosen metric, we have obtained the brane-based version of (43) as given in (49). The cosmological behavior changes depending on values of $C + E^2$ being either elliptic or hyperbolic. It is worth emphasizing that the induced metric on the brane (52) represents the Kasner type spacetime when the fifth coordinate is fixed to zero. However one needs further calculations for the scale factor $R(\tau)$, which requires initial conditions and the matter-energy distribution on the brane have been determined.

V. ACKNOWLEDGEMENTS

-
- [1] Shinji Mukohyama, Tetsuya Shiromizu, Kei-ichi Maeda, Global structure of exact cosmological solutions in the brane world, Phys.Rev. D62 , 024028 (2000)
 - [2] Peter Bowcock, Christos Charmousis, Ruth Gregory, General brane cosmologies and their global spacetime structure, Class.Quant.Grav. 17 , 4745-4764, (2000)
 - [3] V. A. Rubakov and M. E. Shaposhnikov, Do we live inside a domain wall? Phys. Lett. B 125,136 (1983)
 - [4] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, The hierarchy problem and new dimensions at a millimeter, Phys. Lett. B 429, 263, (1998)
 - [5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phenomenology, astrophysics and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity, Phys. Rev.D 59, 086004, (1999)
 - [6] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, New dimensions at a millimeter to a Fermi and superstrings at a TeV, Phys. Lett. B 436, 257, (1998)
 - [7] L. Randall and R. Sundrum, A large mass hierarchy from a small extra dimension, Phys. Rev. Lett. 83, 3370, (1999)
 - [8] L. Randall and R. Sundrum, An Alternative to Compactification, Phys. Rev. Lett. 83, 4690, (1999)
 - [9] Pierre Binetruy, Cedric Deffayet, David Langlois, Non-conventional cosmology from a brane-universe, Nucl.Phys.B565:269-287, (2000)
 - [10] Pierre Binetruy, Cedric Deffayet, Ulrich Ellwanger, David Langlois, Brane cosmological evolution in a bulk with cosmological constant, Phys.Lett.B477:285-291, (2000)
 - [11] Nemanja Kaloper, Bent Domain Walls as Braneworlds, Phys.Rev. D60 123506, (1999)
 - [12] Csaba Csaki, Michael Graesser, Christopher Kolda, John Terning, Cosmology of One Extra Dimension with Localized Gravity, Phys.Lett.B462:34-40, (1999)

- [13] James M. Cline, Christophe Grojean, Geraldine Servant, Cosmological Expansion in the Presence of Extra Dimensions, Phys.Rev.Lett. 83 , 4245 (1999)
- [14] Hang Bae Kim, Hyung Do Kim, Inflation and Gauge Hierarchy in Randall-Sundrum Compactification, Phys.Rev. D61, 064003, (2000)
- [15] O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, Modeling the fifth dimension with scalars and gravity, Phys.Rev.D62: 046008, (2000)
- [16] P. Kanti, I.I. Kogan, K.A. Olive, M. Pospelov, Cosmological 3-Brane Solutions, Phys.Lett.B468:31-39, (1999)
- [17] J. Cline, C. Grojean, G. Servant, Inflating Intersecting Branes and Remarks on the Hierarchy Problem, Phys.Lett. B 472, (2000)
- [18] Dan N. Vollick, Cosmology on a Three-Brane, Class.Quant.Grav. 18, 1-10, (2001)
- [19] Horace Stoica, S.-H. Henry Tye, Ira Wasserman, Cosmology in the Randall-Sundrum Brane World Scenario, Phys.Lett.B 482:205-212, (2000)
- [20] Per Kraus, Dynamics of Anti-de Sitter Domain Walls, JHEP 9912, 011, (1999)
- [21] Tetsuya Shiromizu, Kei-ichi Maeda, Misao Sasaki, The Einstein Equations on the 3-Brane World, Phys.Rev.D62:024012, (2000)
- [22] Daisuke Ida, Brane-world cosmology, JHEP 0009 014, (2000)
- [23] H.A. Chamblin, H.S. Reall, Dynamic Dilatonic Domain Walls, Nucl.Phys.B562: 133-157, (1999)
- [24] Antonio Campos, Roy Maartens, David Matravers, Carlos F. Sopuerta, Braneworld Cosmological Models with Anisotropy, Phys.Rev. D68, 103520, (2003)
- [25] Pantelis S. Apostolopoulos, Nikolaos Tetradis, Brane Cosmology with Matter in the Bulk. I, Class.Quant.Grav. 21, 4781-4792, (2004)
- [26] Laszlo Gergely, Roy Maartens, Brane-world generalizations of the Einstein static universe, Class.Quant.Grav. 19, 213-222, (2002)
- [27] A.Fabbri, D.Langlois, D.A.Steer, R.Zegers, Brane cosmology with an anisotropic bulk, JHEP 0409 :025, (2004)
- [28] Andrei V. Frolov, Kasner's AdS spacetime and anisotropic brane-world cosmology, Physics Letters B 514, 213-216, (2001).
- [29] Mıximo Ba-ados, Claudio Teitelboim, Jorge Zanelli, The Black Hole in Three Dimensional Space Time, Phys.Rev.Lett. 69 1849-1851, (1992)

Appendix A: Field Equations for the Anisotropic Bulk-based Model

$$\frac{f}{2} \left(\frac{a''}{a} + \frac{b''}{b} + \frac{c''}{c} \right) + \left(\frac{f'}{4} + \frac{2f}{\hat{r}} \right) \left(\frac{a'}{a} + \frac{b'}{b} + \frac{c'}{c} \right) - \frac{f}{4} \left(\left(\frac{a'}{a} \right)^2 + \left(\frac{b'}{b} \right)^2 + \left(\frac{c'}{c} \right)^2 \right) + \frac{f}{4} \left(\frac{a' b'}{a b} + \frac{a' c'}{a c} + \frac{b' c'}{b c} \right) + \frac{3f'}{2\hat{r}} + \frac{3f}{\hat{r}^2} = \Lambda_5 \quad (\text{A1})$$

$$\frac{f}{2} \left(\frac{b''}{b} + \frac{c''}{c} \right) + \left(\frac{f'}{2} + \frac{3f}{2\hat{r}} \right) \left(\frac{b'}{b} + \frac{c'}{c} \right) - \frac{f}{4} \left(\left(\frac{b'}{b} \right)^2 + \left(\frac{c'}{c} \right)^2 \right) + \frac{f}{4} \left(\frac{b' c'}{b c} \right) + \frac{f''}{2} + \frac{f'}{2\hat{r}} + \frac{f}{\hat{r}^2} = \Lambda_5 \quad (\text{A2})$$

$$\frac{f}{2} \left(\frac{a''}{a} + \frac{c''}{c} \right) + \left(\frac{f'}{2} + \frac{3f}{2\hat{r}} \right) \left(\frac{a'}{a} + \frac{c'}{c} \right) - \frac{f}{4} \left(\left(\frac{a'}{a} \right)^2 + \left(\frac{c'}{c} \right)^2 \right) + \frac{f}{4} \left(\frac{a' c'}{a c} \right) + \frac{f''}{2} + \frac{f'}{2\hat{r}} + \frac{f}{\hat{r}^2} = \Lambda_5 \quad (\text{A3})$$

$$\frac{f}{2} \left(\frac{a''}{a} + \frac{b''}{b} \right) + \left(\frac{f'}{2} + \frac{3f}{2\hat{r}} \right) \left(\frac{a'}{a} + \frac{b'}{b} \right) - \frac{f}{4} \left(\left(\frac{a'}{a} \right)^2 + \left(\frac{b'}{b} \right)^2 \right) + \frac{f}{4} \left(\frac{a' b'}{a b} \right) + \frac{f''}{2} + \frac{f'}{2\hat{r}} + \frac{f}{\hat{r}^2} = \Lambda_5 \quad (\text{A4})$$

$$\left(\frac{f'}{4} + \frac{f}{\hat{r}} \right) \left(\frac{a'}{a} + \frac{b'}{b} + \frac{c'}{c} \right) + \frac{f}{4} \left(\frac{a' b'}{a b} + \frac{a' c'}{a c} + \frac{b' c'}{b c} \right) + \frac{3f'}{2\hat{r}} + \frac{3f}{\hat{r}^2} = \Lambda_5 \quad (\text{A5})$$

Equations correspond (00), (ii), (44) respectively. The prime denotes the differentiate with respect to \hat{r} .